Key Concepts: Error Analysis

Means, Standard Deviations & Standard Errors:

Unweighted Mean

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Weighted Mean

$$\bar{x}_w = \frac{\sum_i x_i/\sigma_i^2}{\sum_i 1/\sigma_i^2}$$

Unweighted Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

Weighted Standard Error

$$\sigma_{\bar{x}_w} = \left(\frac{1}{\sum_i \frac{1}{\sigma_i^2}}\right)^{\frac{1}{2}}$$

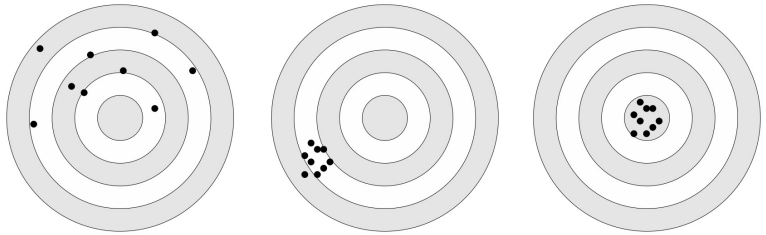
Reporting Uncertainty:

$$\bar{x} \pm \frac{s}{\sqrt{N}}$$

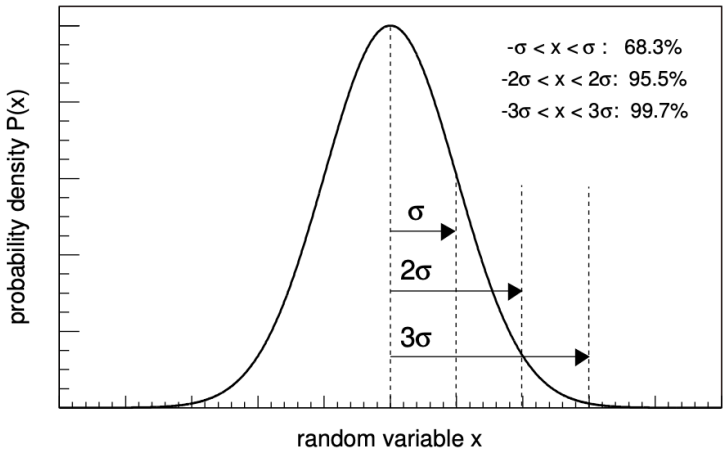
Report with Standard Error of the Mean (aka. σ)

Accuracy vs. Precision:

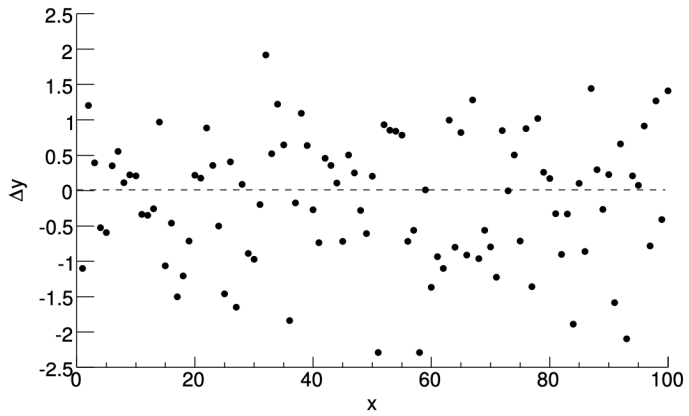
Accuracy: Measured value is close to target value
Precision: Distribution of measured values is small



The Gaussian Distribution:

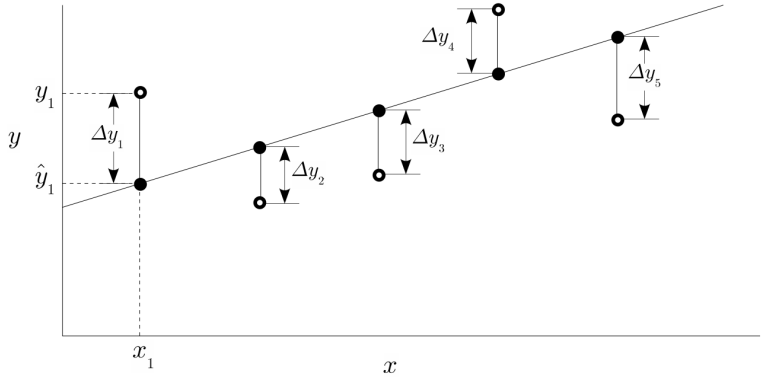


Residuals to Fits: $\Delta y = y - \hat{y}$



Fits & Regressions:

Goal: to minimize → $\chi^2 = \sum_{i=1}^{N} \frac{(y_i - \hat{y}_i)^2}{\sigma_i^2},$



$$y = ax + b,$$

Unweighted Least Squares

$$a = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{D}$$

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{D}$$

$$D = N\sum x_i^2 - \left(\sum x_i\right)^2$$

$$\sigma_a = \sigma\left(\frac{N}{D}\right)^{1/2}$$

$$\sigma_b = \sigma\left(\frac{\sum x_i^2}{D}\right)^{1/2}$$

Weighted Least Squares

$$a_w = \frac{\sum \frac{1}{\sigma_i^2}\sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2}\sum \frac{y_i}{\sigma_i^2}}{E}$$

$$b_w = \frac{\sum \frac{x_i^2}{\sigma_i^2}\sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2}\sum \frac{x_i y_i}{\sigma_i^2}}{E}$$

$$E = \sum \frac{1}{\sigma_i^2}\sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2$$

$$\sigma_{a,w} = \left(\frac{\sum 1/\sigma_i^2}{E}\right)^{1/2}$$

$$\sigma_{b,w} = \left(\frac{\sum x_i^2/\sigma_i^2}{E}\right)^{1/2}$$