Columbia University
Department of Physics
QUALIFYING EXAMINATION

Monday, January 14, 2019
10:00AM to 12:00PM
Classical Physics
Section 1. Classical Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 1 (Classical Mechanics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on Classical Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

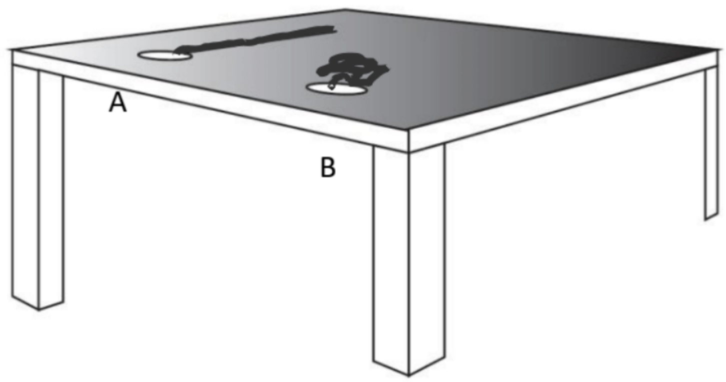
Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1. Consider two uniform ropes (A and B), each of length L, and each of which can pass through holes in a table. A small piece of each rope extends through each of the holes. Each rope is released from rest and starts to fall through its hole in the table. In the case of rope A, the rope starts stretched out along the table and moves as a whole towards the hole. In the case of rope B, the rope starts in a heap near the hole. As rope B unwinds from the heap, the part on the table stays at rest and only the part that has passed through the hole moves downward. Friction does not act on either rope.

   Consider the speeds of rope A and rope B at the instant the ropes lose contact with the table. Are the speeds equal, or is the speed of rope A larger than that of rope B, or vice versa? Prove your answer.



2. A mass m moves in a circular orbit of radius r_0 under the influence of a central force whose potential is given by $V(r) = -\frac{km}{r^n}$, where k is a positive constant and n is a positive integer.

   (a) Determine the constraint on n that must hold for the particle orbit to be stable under small oscillations (i.e., the mass will oscillate about the circular orbit).

   (b) For $n = 1$, compute the frequency of small radial oscillations about this circular orbit.

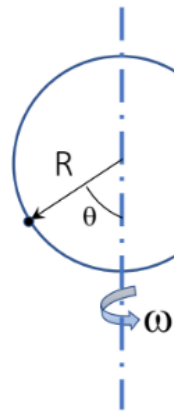
3. A passenger automobile of length L (measured between the rear and front wheels) is driven off a cliff and falls by a vertical distance H. The automobile hits the ground at a distance D away from the cliff.

   Provide an approximate expression for the the number of rotations the automobile would make before it hits the ground in terms of D, H, and L.

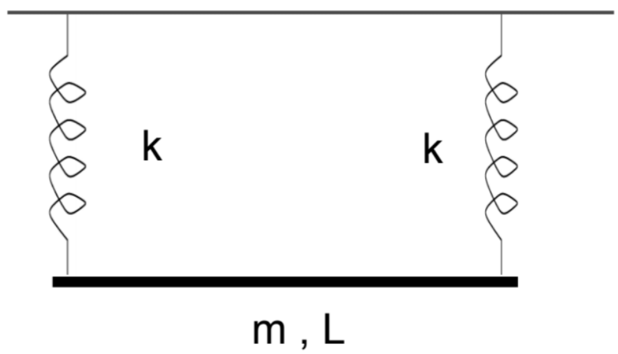
4. A circular wire hoop of radius R spins with fixed angular velocity ω about a vertical axis passing through its center. A point particle of mass m slides without friction along the wire. There is a gravitational acceleration g in the downward direction.

   (a) Find all values of θ where the particle can be stationary with θ fixed and classify their stability for each of the stationary modes.

   (b) For the stable modes, find the frequency for small oscillations.



5. A rod of length L and mass m is supported by two springs of spring constant k at the ends of the rod as shown in the figure. Assuming that the rod remains in the vertical plane and that there is no swinging motion in the horizontal directions, find the normal modes for small oscillations and the oscillation frequency for each mode.



Columbia University
Department of Physics
QUALIFYING EXAMINATION

Monday, January 14, 2019
2:00PM to 4:00PM
Classical Physics
Section 2. Electricity, Magnetism & Electrodynamics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 2 (Electricity etc.), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on Classical Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

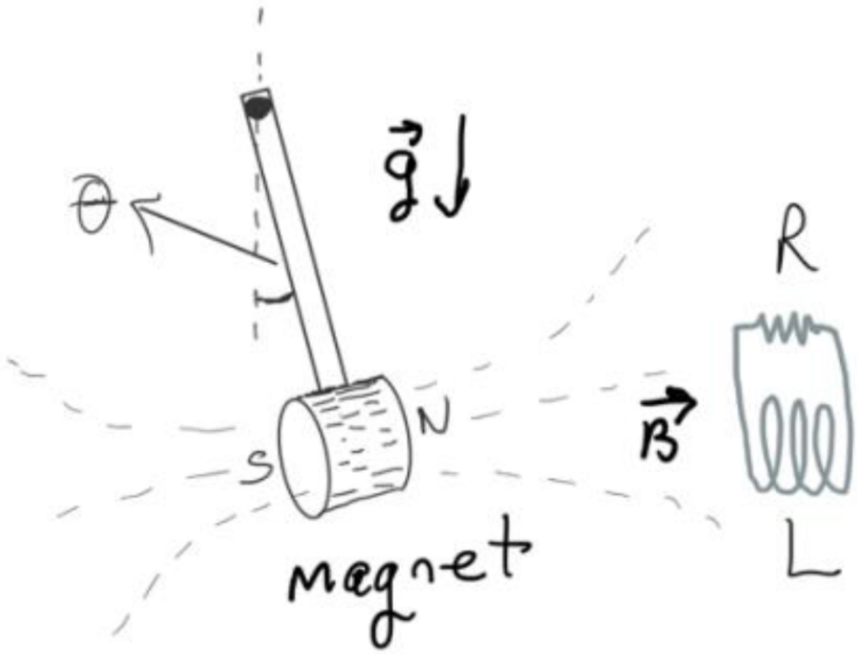
1. Two polarizable atoms, A and B, are separated by a fixed distance z along the z-axis. Atom A is polarized with a dipole moment $\mathbf{p}_A = p_A\hat{\mathbf{z}}$ as a result of the electric field due to the dipole moment $\mathbf{p}_B = p_B\hat{\mathbf{z}}$ of the atom B. Likewise, the dipole moment $\mathbf{p}_B$ is the result of polarization by the electric field due to $\mathbf{p}_A$. No other charges or electric fields are present in the problem other than the two dipoles.

   (a) Are $\mathbf{p}_A$ and $\mathbf{p}_B$ parallel or anti-parallel to one another? Explain.

   (b) Calculate the force between the two electric dipoles in terms of the givens (p_A, p_B, z).

   (c) Is the force calculated in part (b) attractive or repulsive?

2. A magnet of mass m is attached to a rigid rod of length ℓ to create a pendulum as shown in the figure below. The vertical gravitational acceleration is given by g.



A coil of inductance L is brought to the vicinity of the pendulum. The magnetic flux ϕ through the coil changes as the pendulum swings back and forth. ϕ is related to the instantaneous angle of the pendulum $\theta(t)$ by

$$\phi(t) = \phi_a + \phi_b \theta(t).$$

A resistor R is attached to the terminals of the inductance coil so that current flows through the coil in response to the changing magnetic flux.

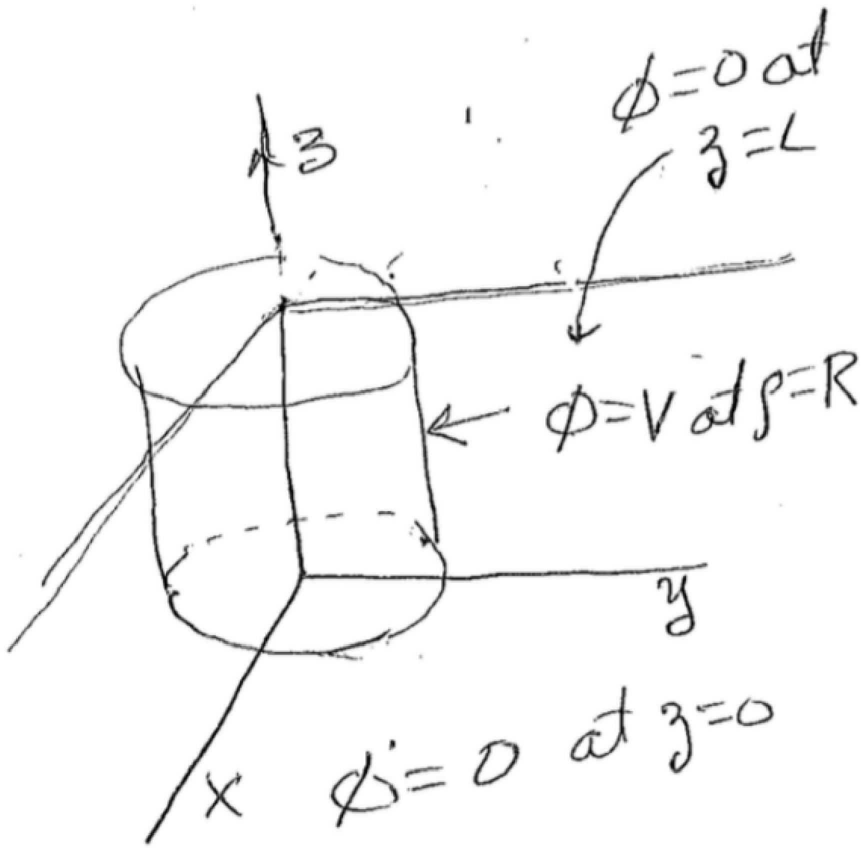
Determine an approximate expression for the amplitude $\Theta(t)$ of the oscillation angle θ in terms of ℓ, R, m, ϕ_b, and the initial amplitude Θ_0.

You can assume $\theta(t)$ has the form $\theta(t) = \Theta(t)\cos(\omega t)$ with Θ small. You can assume that the magnet and coil are weakly couped such that $\Theta(t)$ does not change significantly during a single oscillation cycle, i.e., $\dot{\Theta} << \Theta\omega$, and that the oscillation frequency of the pendulum is not changed significantly by the presence of the coil.

3. Consider a cylinder of radius $\rho = R$ and height $z = L$ as shown in the figure below. The electric potential ϕ on the outer surface of the cylinder is a specified function of height:

$$V(z) = \phi\,(\rho = R, 0 < z < L)\,.$$

The top and bottom of the cylinder are bounded by infinite sheets (the surfaces $z = 0$ and $z = L$, respectively) held at zero electric potential $\phi = 0$.



Find a solution for the electric potential $\phi(\rho, z)$ in the region $(\rho > R, 0 < z < L)$, i.e., outside the cylinder and between the plates. Assume that no charge resides in this region. You may express your result in terms of special functions.

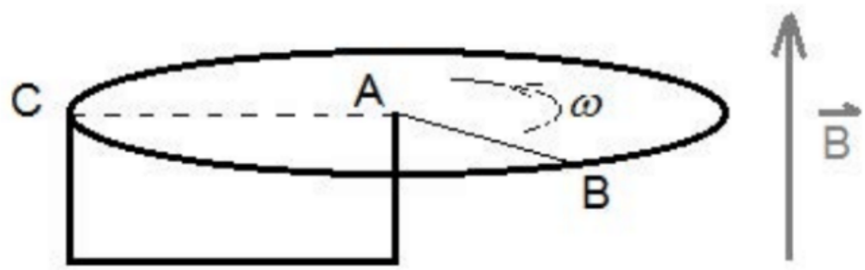
Hints: The Laplacian of a scalar function f is expressed in cylindrical coordinates as

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}.$$

The modified Bessel functions $K_n(x)$ satisfy this differential equation:

$$\frac{d^2 K_n(x)}{dx^2} + \frac{1}{x}\frac{dK_n(x)}{dx} - \left(1 + \frac{n^2}{x^2}\right) K_n(x) = 0.$$

4. A horizontal ring of radius r is placed in a uniform vertical magnetic field $\mathbf{B}$. A bar of resistance R connects the center of the ring (point A in the figure) to a point B on the ring. The bar rotates about A, moving with constant angular velocity ω. The angular velocity of the point B is parallel to the magnetic field $\mathbf{B}$. Point A is also connected to the ring by a stationary wire at the point C. Both the ring and the wire joining points A and C have negligible resistance. The moving bar makes frictionless electrical contact at each of its ends.

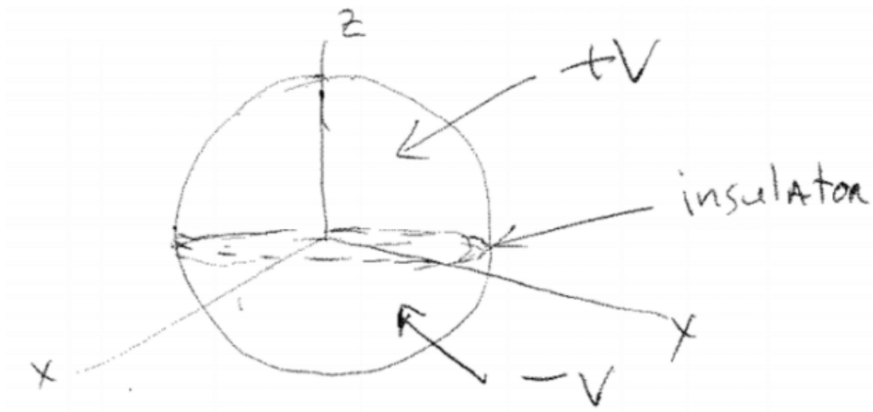


(a) Find the current flowing in the bar joining the points A and B.

(b) Find the rate of energy loss in the moving bar due to Ohmic heating.

(c) Compare your answer to part (b) with the work needed to rotate the wire.

5. Consider a hollow conducting sphere of radius a made up of two hemispherical sections separated by a thin insulating ring. The upper hemisphere is held at potential $+V$ and the lower hemisphere at potential $-V$.



(a) Find expressions for the interior potential ($r < a$) and the exterior potential ($r > a$) in an appropriate series expansion. Your final answer may include unevaluated definite integrals over special functions.

(b) Explicitly evaluate the two lowest order terms for the interior and exterior potential.

You may find the following properties of the Legendre polynomials useful:

$$P_n(-x) = (-1)^n P_n(x)$$

$$\int_{-1}^{1} P_m^2(x)\, dx = \frac{2}{2m+1}.$$

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Wednesday, January 16, 2019
10:00AM to 12:00PM
Modern Physics
Section 3. Quantum Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 3 (QM), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on Modern Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1. Consider a free particle in one dimension with mass m. At time $t = 0$ the expectation value of its position is $\langle x \rangle_0$, with a variance $(\Delta x)_0^2 = \langle x^2 \rangle_0 - \langle x \rangle_0^2$. Find the variance at some later time t. Express your answer in terms of t and expectation values of operators at $t = 0$, including the operators x, p, and combinations thereof.

2. Consider a particle in one dimension subject to a harmonic potential. The Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{1}{2} m\,\omega^2\, x^2.$$

Suppose that for $t < 0$ the particle is in the ground state of H. At $t = 0$, the particle is suddenly "liberated", in the sense that the potential is very quickly tuned to zero, and from that moment on the particle behaves as a free particle.

(a) What is the state of the particle right after being liberated?

(b) In order to tune the potential to zero, the experimenter had to do work on the particle. How much work? Which sign?

(c) Show that for very large positive times, $t >> 1/\omega$, the probability density to find the particle at a generic position x becomes flat, that is, x-independent.

3. Use trial wavefunctions of the form

$$\psi(x) = \left(\frac{a}{\pi}\right)^{1/4} e^{-a^2x^2/2}$$

(where a is an arbitrary constant) to estimate, and bound, the ground state energy of the one-dimensional Hamiltonian

$$H = \frac{p^2}{2m} - V_0\delta(x),$$

where V_0 is a constant and $V_0 > 0$.

4.

(a) Consider a free particle in one dimension with Hamiltonian $H = \frac{1}{2}p^2 = -\frac{1}{2}\partial_x^2$. Gaussian wave packets stay Gaussian wave packets under time evolution. Find the time evolution of the wave function

$$\psi(t, x) = e^{-\frac{x^2}{2A(t)} - C(t)}$$

with initial condition $A(0) = 0$, which is to say the particle is initially sharply localized at $x = 0$.

(b) Now consider a particle in a constant gravitational field, $H = \frac{1}{2}p^2 + g\,x$. This Hamiltonian still has the property that Gaussian wave packets evolve into Gaussian wave packets, but now the ansatz has to be generalized slightly to

$$\psi(t, x) = e^{-\frac{x^2}{2A(t)} - B(t)\,x - C(t)}.$$

Find the time evolution with initial conditions $A(0) = B(0) = 0$.

(c) Take the solution obtained in part (b) and replace t by $t - i$ (i as in $i = \sqrt{-1}$). The resulting wavefunction is still a perfectly good solution. Can you explain why? (i.e. What symmetry tells you a new solution can be obtained this way?) At what x does the resulting $|\psi|^2$ peak? (It would be some function of time).

5. Consider a particle of mass m moving in one spatial dimension (x) with a potential of the form of a square barrier, i.e. the potential equals V for $0 < x < L$, but is zero everywhere else. Here $V > 0$. The Hamiltonian is the usual kinetic energy $p^2/(2m)$ plus the potential. Suppose the particle is coming in from $x = -\infty$ as a plane wave, i.e. the incoming wave-function takes the form $e^{-iEt+ikx}$, with $E = k^2/(2m)$. For $E < V$, it can be shown that the probability of reflection by the barrier is

$$P_{\text{reflection}} = \frac{1}{1 + \frac{4E(V-E)}{V^2}\frac{1}{[\sinh\alpha]^2}}.$$

What is α in terms of the quantities given in the problem?

Note: This problem can be solved exactly, but α can be found much more easily by matching with an approximation for $P_{\text{reflection}}$ that is valid when $V >> E$.

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Wednesday, January 16, 2019
2:00PM to 4:00PM
Modern Physics
Section 4. Relativity and Applied Quantum Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 4 (Relativity and Applied Quantum Mechanics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on Modern Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1. A cosmic ship with rest-mass m is moving with speed $v = 0.6c$ with respect to an inertial "lab" frame. No external forces act on the ship. At time $t = 0$ the ship switches on a laser with power P_0 and emission frequency ν_0 (both measured in the rest frame of the ship). The laser beam propagates ahead of the ship, in the direction of its motion.

   (a) What is the laser emission frequency ν measured in the lab frame at $t = 0$?

   (b) What is the laser emission power P measured in the lab frame at $t = 0$?

   (c) The laser continues to radiate power $P_0 = \text{const.}$, and recoil from the emitted photons decelerates the ship. Find the mass of the ship when it stops in the lab frame.

   (d) Find the proper time τ (measured by the clock on the ship) after which the ship stops in the lab frame.

2. A space explorer leaves Earth in a ship and accelerates at 3 m/s^2 for one year. She then coasts for one year, and spends a third year decelerating at the same rate as the original acceleration ("accerlerates for one year", "coasts for one year" and "spend a third year" all refer to "one year" in the explorer's reference frame). How far from Earth did the explorer get? (Take 1 year $= 3.2 \times 10^7$ s, $c = 3 \times 10^8$ m/s.)

   Here, constant acceleration at some rate a (e.g. 3 m/s^2) means this:

$$-\left(\frac{d^2t}{d\tau^2}\right)^2 + \frac{1}{c^2}\left(\frac{d^2x}{d\tau^2}\right)^2 = \frac{a^2}{c^2},$$

   where t and x are the time and distance according to the Earth frame, and τ is the proper time of the space traveler i.e.

$$-\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{c^2}\left(\frac{dx}{d\tau}\right)^2 = -1.$$

3. A proton of energy E collides with a neutron at rest.

   (a) Calculate the threshold energy E for neutral pion production: $p + n \rightarrow p + n + \pi^0$. You should take the masses of the proton and neutron to be equal. Express your answer in terms of the masses of the particles.

   (b) The neutral pion then decays into two photons: $\pi^0 \rightarrow \gamma + \gamma$. Calculate the energy of a photon produced in such a decay. Express your answer in terms of the masses of the particles.

   You may assume that the photons are emitted parallel and anti-parallel, respectively, to the momentum of the pion in the lab frame. Explain why this will maximize the energy of the parallel photon.

4. One electron is moving in the weak periodic potential of a linear chain along the x axis, with sites located at positions given by

$$x_n = na,$$

where $n = 0, 1, 2, ..., N = L/a \to \infty$, where L is the total length of the chain. The chain has a weak periodic potential, which can be expanded in a Fourier series.

$$V(x) = \sum_m V_m e^{iK_m x} = V(x+a),$$

where $K_m = 2\pi m/a$ and $m = 1, 2, 3, ...$

The wavefunction of the electron states can be expanded as a set of plane waves

$$\psi(x) = \sum_q C(q) e^{iqx},$$

where the expansion coefficients $C(q)$ can be evaluated from the Schroedinger equation using the potential $V(x)$. Hereafter just assume the $C(q)$ are given.

(a) We are interested in the symmetry of the wavefunctions. Writing

$$q = K_m + k,$$

prove that the wave function for states with wave vector k can be written as

$$\psi_k(x) = e^{ikx} u_k(x), \tag{1}$$

i.e. find an expression for $u_k(x)$.

(b) Show that

$$u_k(x) = u_k(x+a). \tag{2}$$

(c) Show that the wave functions given by equations (1) and (2) satisfy

$$\psi_k(x) = \psi_{k+K_m}(x) \tag{3}$$

(d) Equation (3) implies that the energy eigenvalues satisfy:

$$E(k) = E(k+K_m) \tag{4}$$

so that the energy states form a set of bands linked to the index m, i.e., for each m there is an interval of length $\Delta k = 2\pi m/a$ of k values that give distinct energies.

For the lowest band ($m = 0$)

$$E(k) \simeq \frac{\hbar^2 k^2}{2m_0}, \tag{5}$$

where m_0 is the free electron mass.

An electric field is applied. In the so-called semi-classical approximation, the equation for the motion of the electron is given by

$$\hbar \frac{dk}{dt} = -eE \tag{6}$$

Assume that the electron moves only within the next lowest band ($m = 1$). As the electron moves through the states of the band, the electron energy linked to changes in the wave vector k changes periodically as a function of time.

Calculate the frequency of the periodic variation in the electron energy.

5. A beam of electrons with random spins is passed along the y-axis through a Stern-Gerlach apparatus blocking $s_z = -\frac{1}{2}$ states $|{\downarrow}\rangle$, allowing only $s_z = +\frac{1}{2}$ states $|{\uparrow}\rangle$ to pass through.

   (a) **What fraction of the initial beam gets through this spin filter?**
   Let us call this filter A.

   (b) A second Stern-Gerlach apparatus—let us call it filter B—is placed in sequence after filter A, but rotated by 180 degrees around the beam axis so only $s_z = -\frac{1}{2}$ states $|{\downarrow}\rangle$ are allowed through.

   What fraction of the initial beam gets through both filters A and B?

   (c) Another Stern-Gerlach filter (C) is placed *between* filter A and B, now rotated by 90 degrees around the beam axis so only $s_x = +\frac{1}{2}$ state $|{\rightarrow}\rangle$ are allowed through.

   What fraction of the initial beam gets through the full sequence of three filters $A \to C \to B$?

   (d) More generally, say we have a sequence of two subsequent filters rotated by an angle ϕ along the beam axis with respect to each other.

   What fraction of the beam that survived the first filter also survives the second filter?

   (e) Finally, consider a setup in which N filters are placed between A and B, each rotated with respect to the previous one by an angle $\phi = \dfrac{\pi}{N+1}$.

   What fraction of the beam passes through the full sequence of filters in the limit $N \to \infty$?

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 18, 2019
10:00AM to 12:00PM
General Physics
Section 5. Thermodynamics and Statistical Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 5 (Thermodynamics and Statistical Mechanics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on Modern Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1.

(a) Using dimensional analysis, find the thermal energy U of an ultra-relativistic ideal gas (such as a photon gas) in a box of volume V at a temperature T, up to an overall constant.

(b) Derive from the result in part a the dimensionless ratios CT/U, F/U and PV/U, where C is the heat capacity at constant volume, $F = U - TS$ is the free energy, and P is the pressure.

(c) By what factor does the temperature increase when the volume of the box is adiabatically compressed to half its original volume?

2. Consider a long molecule which is composed of N ($N >> 1$) monomers, each of which can reside in one of two states which have lengths a and b, with $b > a$. The energy of a monomer in the longer state is ε larger than a monomer in the shorter state (i.e. $\Delta E = \varepsilon$).

   (a) Calculate the mean length of the molecule as a function of temperature, T.

   (b) Calculate the standard deviation of the molecules' length.

   Now, suppose the molecule is held at fixed length, L, where $Na < L < Nb$.

   (c) Find the internal energy and entropy for the molecule.

   (d) Calculate the force needed to keep the molecule held at length L.

3. In addition to the cosmic microwave background (CMB) of photons, the universe is permeated with an analogous background radiation of neutrinos, currently at an effective temperature of 1.95 K. In the hot early universe, neutrinos were in thermal equilibrium with photons: neutrino-antineutrino pairs were freely converted into photons and back. For the purposes of this problem, you may assume neutrinos are massless. Find the present total number of cosmic neutrinos plus anti-neutrinos per cubic meter, assuming there are three different neutrino species. (A precision of one significant digit suffices in your final result.)

   The following integral formula may be useful:

$$\int_0^\infty dx\, \frac{x^n\, e^{-x}}{1 - a\, e^{-x}} = \sum_{k=0}^{\infty} \int_0^\infty dx\, x^n\, a^k\, e^{-(k+1)x} = n! \sum_{k=0}^{\infty} \frac{a^k}{(k+1)^n}. \tag{1}$$

   Boltzmann's constant is $k = 8.6 \times 10^{-5}\,\mathrm{eV/K}$. Planck's constant is $\hbar = 6.6 \times 10^{-16}\,\mathrm{eV \cdot s}$.

4. Consider two containers. One is of volume V_1 and contains N_1 oxygen atoms in thermal equilibrium at temperature T. The other is of volume V_2 and contains N_2 atoms of nitrogen, also in thermal equilibrium at temperature T. Suppose that the two containers are now connected so they can exchange particles and we have one system of volume $V = V_1 + V_2$ at temperature T.

   (a) Please find the pressure.

   (b) Please find the entropy.

   (c) Please state how your answers would change if both containers held oxygen initially.

5. Suppose that we have a gas of noninteracting particles moving in two dimensions with the dispersion

$$\varepsilon_k = v\sqrt{k_x^2 + k_y^2}$$

Please determine the following. *You may express your answers in terms of physical constants and integrals such as*

$$I(\zeta) = \int_0^\infty dx \, \frac{x^\zeta}{e^x - 1},$$

which evaluate to pure numbers. You do not need to give numerical values for the integrals.

(a) the Bose-Einstein condensation temperature T_{BEC}

(b) the specific heat at constant volume for temperatures $T < T_{BEC}$

(c) the behavior of $dN/d\mu$ as the temperature approaches T_{BEC} from above.

6. Consider an ideal gas composed of N ^{3}He atoms contained in a vessel of volume V. The ^{3}He isotope has a nucleus consisting of two protons and one neutron. In the following, consider energy states of the ^{3}He atoms in a cubic box of volume $V = L^3$, where L is the length of the sides of the cube, and apply cyclic boundary conditions (*In cyclic boundary conditions the wavefunctions are unchanged under a translation of distance L along any coordinate).* Assume, for simplicity, that the system remains gaseous at all temperatures and assume that the ^{3}He atoms remain in their electronic ground state.

   (a) Consider the limit of absolute zero of temperature ($T = 0$).

      (i) Provide an expression specifying the possible energy states for single ^{3}He atoms.

      (ii) What is the difference between the lowest and highest energy states of single ^{3}He atoms?

      (iii) Obtain an expression for the total kinetic energy as function of N and V.

      (iv) What would be the total kinetic energy for a gas composed of ^{4}He atoms?

   (b) Assume that the temperature is raised slightly so that T remains small. The kinetic energy of the ^{3}He ideal gas can be written:

$$U(T) = U(0) + u(T),$$

   where $u(T)$, to leading power in the temperature is proportional to T^n. Using qualitative and/or dimensional arguments, determine n.

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 18, 2019
2:00PM to 4:00PM
General Physics
Section 6. Various Topics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 6 (General Physics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on Modern Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

Good Luck!

1. The spectrum of a diatomic molecule includes transitions between (1) electronic levels, between (2) vibrational levels, and between (3) rotational levels.

   (a) Give order of magnitude estimates for each of these transition energies (1)-(3) in terms of nuclear and electron masses and other fundamental constants.

   (b) Order the transitions (1)-(3) from lowest to highest energy.

   (c) To approximately what parts of the electromagnetic spectrum do these transitions correspond?

2. What is the critical angle for total external reflection for photons of wavelength λ and frequency $\omega = 2\pi c/\lambda$ in vacuum, falling on a metal plate with electron density N? Assume that electrons in a metal are essentially free.

3. The radius of the event horizon of a (static) black hole of mass M may be estimated using Newtonian methods as the radius where the escape velocity reaches the speed of light. Objects crossing the event horizon disappear from view, but may experience large tidal forces while falling into the black hole.

   Estimate the mass of a black hole sufficiently large that a Sun-sized star would not be disrupted when crossing the event horizon. Express your answer as a multiple of the solar mass $M_\odot$. Take the radius of the Sun to be $R_\odot = 7 \times 10^5\,\mathrm{km}$, and use the fact that if it were to form a black hole its event horizon would be about $R_S \sim 3\,\mathrm{km}$.

4. A plane-parallel atmosphere in a uniform vertical gravitational field g is illuminated from above by a vertical radiation flux F [$\mathrm{erg} \cdot \mathrm{s}^{-1} \cdot \mathrm{cm}^{-2}$]. The atmosphere is isothermal, with a temperature T. It is composed of particles of mass m, which isotropically scatter radiation with cross section σ. The atmosphere is optically thin. The surface below it, at $z = 0$, absorbs the impinging radiation flux. The particle number density at $z = 0$ is n_0.

   (a) Find the vertical distribution of the atmosphere density $n(z)$ at $z > 0$.

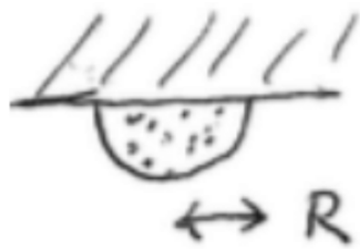
   (b) How will $n(z)$ change if the surface $z = 0$ reflects radiation rather than absorbs it?

5.

(a) The "surface tension" of water (σ) is the extra energy needed for a unit area increase of the surface between liquid water and an adjacent atmosphere. Use the information below to make a rough estimate of σ.

- Heat of vaporization of liquid water $= 2 \times 10^{10}$ ergs/cm^3
- Mass of a water molecule $= 3 \times 10^{-23}$ g
- Density of water $= 1$ g/cm^3

(b) Use σ (and any other natural constants you might need) to estimate the maximum radii of water droplet hemispheres, which may still be clinging to the outside flat horizontal bottom of a newly washed (but not yet dry) glass.



6.

(a) Estimate the mass of the sun from your knowledge of its distance and any other common facts.

(b) Estimate the radius of the sun from common sense observations.

(c) Estimate the mean interior pressure of the sun.

(d) Estimate the mean interior temperature of the sun.

(e) Estimate the mean energy of a proton in the sun.

(f) Estimate the proton-proton Coulomb barrier that must be exceeded in two protons are to initiate a fusion reaction.

(g) Considering the answers to (e) and (f), how is fusion able to proceed in the sun?