Columbia University
Department of Physics
QUALIFYING EXAMINATION

Monday, January 8, 2018
10:00AM to 12:00PM
Classical Physics
Section 1. Classical Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 1 (Classical Mechanics), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

You may refer to the single handwritten note sheet on $8\frac{1}{2}$" $\times$ 11" paper (double-sided) you have prepared on Classical Physics. The note sheet cannot leave the exam room once the exam has begun. This note sheet must be handed in at the end of today's exam. Please include your Exam Letter Code on your note sheet. No other extraneous papers or books are permitted.

Simple calculators are permitted. However, the use of calculators for storing and/or recovering formulae or constants is NOT permitted.

Questions should be directed to the proctor.

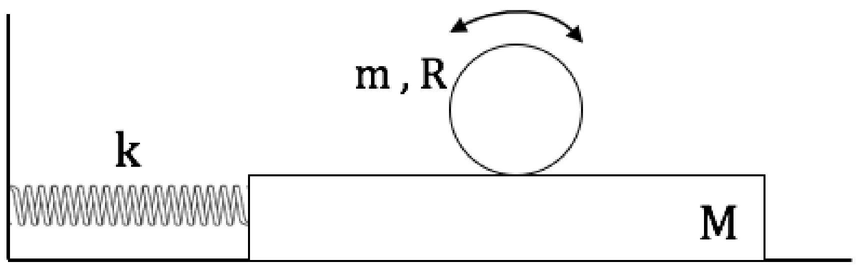
Good Luck!

1. A block of mass M slides along a frictionless surface. The block is connected to the wall with a spring of spring constant k. A cylinder of mass m, radius R, and rotational inertia $\frac{1}{2}mR^2$ rolls without slipping on the block.

   (a) What is the frequency for small oscillations of the system around the starting position?

   (b) Describe the motion associated with this oscillation.

   (c) What is the maximum oscillation amplitude that the block can have before the cylinder starts slipping if the friction coefficient is μ?



2. A particle of mass m moves in a potential energy

$$U = -\frac{k}{r} - \frac{\alpha}{2r^2}$$

where $k > 0$. The particle has angular momentum ℓ.

(a) For what values of α are circular orbits possible? For what values of α are these stable?

(b) Consider an orbit that is almost circular, with r varying between two values $r_{\mathbf{min}}$ and $r_{\mathbf{max}}$. Let T be the time interval required for r to move from $r_{\mathbf{min}}$ to $r_{\mathbf{max}}$. For which values of α is T greater than the period of revolution about the origin (i.e., the time required for ϕ to range from 0 to 2π)? For which values of α is T less than the period of revolution about the origin?

3. A particle of mass m moves in the potential

$$V = \lambda_0 xy + \frac{1}{4}\lambda_1(x^4 + y^4)$$

with λ_0 and λ_1 positive.

(a) When are the stable equilibria for the particle?

(b) Give the Lagrangian appropriate for small oscillations about one of the equilibrium positions.

(c) Give the normal frequencies and modes of vibration corresponding to the equilibrium position in (b).

4. A circular platform with radius R and moment of inertia I_P rotates in the horizontal plane on a frictionless bearing. A bead of mass m and negligible size is free to slide without friction on a wire of length $2R$ along a diameter of the platform.

   (a) At $t = 0$, the bead is released at rest from $r_0 \equiv r(t = 0) = 0.10R$. An external mechanism maintains the platforms angular velocity at a fixed value $\omega_0 = 6.0s^{-1}$. Find the time in seconds for the bead to reach the edge of the platform. It is useful to recall that $e^3 \approx 20$.

   (b) Now assume that at the moment the bead is released the external mechanism is switched off. Find an algebraic expression for the radial velocity of the bead at $r = R$, again assuming it is released at rest from r_0.

5. Consider a particle of charge e and mass m moving under the action of an isotropic harmonic oscillator of potential $U = \frac{1}{2}K(x^2 + y^2 + z^2)$, with K a positive constant. The particle is in a magnetic field aligned along the z-axis $\vec{B} = B\hat{z}$. Assume that the magnetic field is weak, so that $\frac{eB}{2m} \ll \sqrt{\frac{K}{m}}$.

   (a) With the above approximation, find the eigenmodes or normal modes for the motion of the particle and show your solutions correspond to circular motion. What is the angular frequency in your solutions?

   (b) Show explicitly from your solutions that when $B \neq 0$ the particle's motion will generally exhibit precession. What is the precessional frequency?

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Monday, January 8, 2018
2:00PM to 4:00PM
Classical Physics
Section 2. Electricity, Magnetism & Electrodynamics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 2 (Electricity etc.), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

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Questions should be directed to the proctor.

Good Luck!

1. An otherwise free non-relativistic charged particle having mass m and charge e moves in a uniform magnetic field $\vec{B}$ pointing in the $\hat{z}$ direction.

   (a) Assume that at $t = 0$ the particle is located at the origin and moving with velocity $\vec{v}_0$ in the x-direction: $\vec{v}_0 = v_0\hat{x}$. Determine the particle's subsequent position $\vec{r}(t)$ and velocity $\vec{v}(t)$ as a function of time and describe the resulting motion (ignoring radiation damping).

   (b) If the initial velocity $\vec{v}_0$ has both an x- and a z-component, $\vec{v}_0 = v_{0x}\hat{x} + v_{0z}\hat{z}$ find the subsequent postion $\vec{r}(t)$ and describe the resulting motion.

2. A crystal is composed of a collection of identical atoms. The atoms are modeled as point particles with charge q and mass m coupled with spring constant k to fixed atomic sites of opposite charge $-q$. The number density of atoms in this crystal is N. Damping effects are accounted for by the damping constant Γ. A plane polarized electromagnetic wave of frequency ω propagates in the solid. Use the following notation: $\omega_0^2 = k/m$.

   (a) Derive an expression for the complex dielectric function $\epsilon(\omega)$.

   (b) Sketch the real and imaginary parts of $\epsilon(\omega)$.

3. You have a very long ideal solenoid with radius R, N turns per unit length, and current I. Coaxial with the solenoid are two long cylindrical shells of length l. One is inside the solenoid at radius $a < R$ and carries a charge $+Q$ uniformly distributed over its surface. The other is outside the solenoid at radius $b > R$ and carries charge $-Q$ uniformly distributed over its surface. Note that $l >> R$ and ignore fringe fields.

   (a) What is the angular momentum of this system?

   (b) As the current in the solenoid is gradually reduced to zero, the cylinders begin to rotate. Calculate the final angular momentum of each and show that their sum is equal to the initial angular momentum of the system.

4. Consider in electrostatics an infinitely extended uniform charge density, $\rho(\vec{x}) = \rho_0$.

   (a) Prove that, despite $\rho(\vec{x})$ being invariant under translations and rotations, there is no unique solution to the Poisson equation that is preferred in terms of symmetries.

   (b) Suppose now that the charge distribution is smoothly cut off in a spherical fashion at very large distances, i.e.

$$\rho(\vec{x}) = \rho_0 f(|\vec{x}|/R) \ ,$$

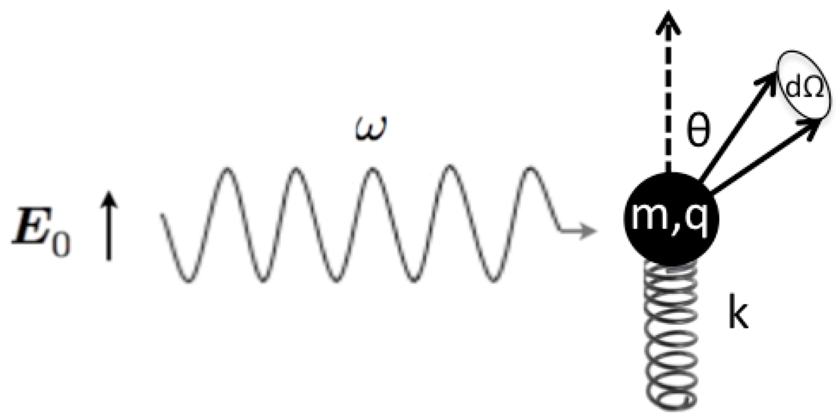
   where $f = 1$ for $|\vec{x}|/R \ll 1$, and $f = 0$ for $|\vec{x}|/R \gg 1$. Argue that this removes the ambiguity in the Poisson problem. Compute the potential for $|\vec{x}|/R \ll 1$ and for $|\vec{x}|/R \gg 1$ (up to additive constants).

5. A linearly polarized electromagnetic wave of amplitude $\vec{E}_0$ and frequency ω is incident on a classical particle of mass m and charge q attached to a spring of spring constant k, as shown in the figure below. Assume that the particle is free to move in three dimensional space, i.e. it is mechanically an isotropic 3-D classical harmonic oscillator.

   (a) Calculate the differential cross section $d\sigma/d\Omega$ for the light to scatter into a solid angle $d\Omega$. Express your results in terms of the angle θ with respect to the polarization direction.

   (b) Calculate the total cross section.

   (c) Consider the above system as a semi-classical model for a valence electron of a nitrogen atom in the Earth's atmosphere, where the "spring" represents the restoring force holding the electron at its equilibrium position in the atom.
   Using your result from part 2, explain why the sunset is red. Make sure to justify any approximations made.



Columbia University
Department of Physics
QUALIFYING EXAMINATION

Wednesday, January 10, 2018
10:00AM to 12:00PM
Modern Physics
Section 3. Quantum Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 3 (QM), Question 2, etc.).

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Good Luck!

1. Let $|0\rangle$ be the ground state of the harmonic oscillator with angular frequency ω, with Hamiltonian $H = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$. The ground state wave function in position representation is $\psi_0(x) = \langle x|0\rangle = A\, e^{-x^2/2\ell^2}$ with $\ell = \sqrt{\frac{\hbar}{m\omega}}$ and $A = \ell^{-\frac{1}{2}}\pi^{-\frac{1}{4}}$

   (a) Express the operator $\hat{x}$ in the *momentum* representation and verify the canonical commutation relation.

   (b) Using the above, express the creation and annihilation operators $\hat{a}$ and $\hat{a}^\dagger$ in the momentum representation, and verify that the canonical commutation relation between them holds.

   (c) Using (b), calculate the ground state wave function $\psi_0(p) = \langle p|0\rangle$ in the momentum representation.

   (d) Check your answer by direction calculation starting from the position space representation $\psi_0(x)$.

2. For the infinite square well with walls located at $x = a$ and $x = -a$, the ground state energy is $E_1 = \pi^2\hbar^2/8ma^2$ and the ground state wavefunction is $\psi_1(x) = \frac{1}{\sqrt{a}}\cos(\pi x/2a)$. The position momentum uncertainty relationship for this state is $\Delta x \Delta p = \kappa\hbar/2$. Find κ.

   (A potentially useful formula is $\int dx\, x^2 \cos^2(bx) = \frac{x^3}{6} + \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin(2bx) + \frac{x\cos(2bx)}{4b^2}$.)

3. Consider the two potential energy functions:

$$
\begin{aligned}
V_1(x) &= \frac{m\omega^2}{2}x^2, \qquad x > 0 \\
&= \infty, \qquad x \leq 0 \\
V_2(x) &= \frac{m\omega^2}{2}x^2, \qquad -\infty \leq x \leq \infty
\end{aligned}
$$

A particle with mass m, subject to the potential V_1, is initially in its quantum ground state.

(a) Write the normalized wave function, $\phi(x)$, of this state. (Hint: This wave function can be determined easily from simple harmonic oscillator solutions you already know.)

(b) At a certain time (say, $t = 0$), the impenetrable barrier at $x = 0$ is *very suddenly* removed so that for all $t > 0$ the same system is subject to the potential V_2 instead of V_1. If the energy is then measured, what is the probability of finding the system in its new ground state, $\psi_0(x)$?

4. In this problem we consider a one-dimensional quantum harmonic oscillator with frequency ω and mass m. The Hamiltonian of the system is

$$\hat{H} = \hbar\omega(\hat{n} + 1/2).$$

The orthonormal eigenstates of the Hamiltonian are denoted by $|n\rangle$ and have the property that $\hat{n}|n\rangle = n|n\rangle$ where $\hat{n} = \hat{a}^\dagger\hat{a}$.

The eigenstates of the operator $\hat{a}$ are called *quasi-classical* states for reasons that we examine in this problem. Consider an arbitrary complex number α.

(a) Show that the state

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

is an eigenstate of $\hat{a}$ with eigenvalue α, that is $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$. Also show that it is properly normalized.

(b) Calculate the expectation value of energy $\langle\hat{H}\rangle = \langle\alpha|\hat{H}|\alpha\rangle$ for a quasi-classical state $|\alpha\rangle$.

(c) Also calculate the expectation values of position $\langle\hat{x}\rangle$ and momentum $\langle\hat{p}\rangle$. Recall that

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger) \quad \text{and} \quad \hat{p} = -i\sqrt{\frac{m\hbar\omega}{2}}(\hat{a} - \hat{a}^\dagger).$$

(d) Calculate the root mean square deviations Δx and Δp for the position and the momentum in this state. Show that $\Delta x \Delta p = \hbar/2$.

(e) Now, suppose that at time $t = 0$ the oscillator is in a quasi-classical state $|\alpha\rangle$ with $\alpha = |\alpha|e^{i\phi}$, where $|\alpha|$ is a real positive number. Show that at any later time t the oscillator is also in a quasi-classical state that can be written as $e^{-i\omega t/2}|\alpha(t)\rangle$. Determine the value of $\alpha(t)$ in terms of $|\alpha|$, ϕ, ω, and t.

(f) Evaluate $\langle\hat{x}\rangle(t)$ and $\langle\hat{p}\rangle(t)$. Justify briefly why these states are called *quasi-classical*.

5. Consider a particle in one dimension subject to the double-delta-function potential energy

$$V(x) = -g\,\delta(x-a) - g\,\delta(x+a)$$

where g is a positive constant.

(a) Find an equation from which the energy of the lowest bound state can be determined.

(b) Using this equation, find approximate expressions for the ground state energy in the limits where a is very large and where a approaches zero.

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Wednesday, January 10, 2018
2:00PM to 4:00PM
Modern Physics
Section 4. Relativity and Applied Quantum Mechanics

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 5 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 4 (Relativity and Applied QM), Question 2, etc.).

Do **NOT** write your name on your answer booklets. Instead, clearly indicate your **Exam Letter Code**.

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Good Luck!

1. Solve both parts of this problem.

   (a) A π^- particle stops in Deuterium and is captured from the atomic ground state creating a pair of outgoing neutrons. Examine the possible combinations of spin and orbital angular momentum of the pair of neutrons and explain why the existence of this process implies that the parity of the π^- must be negative. (Recall that the Deuteron has spin one.)

   (b) A π^- with total energy E_π strikes a hydrogen target producing a Λ^0 and a K^0:

$$\pi^- + P \to \Lambda^0 + K^0.$$

   What is the minimum energy E_π that will allow this reation to take place? (Recall $M_{\pi^-} =$ 140 MeV, $M_{\text{proton}} = 938$ MeV, $M_{\Lambda^0} = 1116$ MeV and $M_{K^0} = 498$ MeV.)

2. A train rushes by a platform moving to the right at speed $v = 12c/13$. Call the midpoint of the train and the midpoint of the platform the "origin" of each frame, and note that as the respective origins pass one another, clocks at those locations both read 4 PM. As the train's conductor, who is sitting in the engine room (at the front of the train), passes an observer on the platform, there's an explosion in the conductor's control console.The platform observer notes that when the explosion happens, a clock in the control room reads 2 PM.

   (a) According to those on the train, what is the reading on the clock at the origin of platform when the explosion takes place?

   (b) According to those on the train, how far is the origin of the platform from the origin of the train frame when the explosioni takes place?

   (c) The condcutor immediately sends a distress signal (traveling at light speed) alerting his colleagues of the explosion. At what time does his colleague at the origin of the train receive the message?

   (d) When the observer at the origin of the train receives the signal, how far away will he/she claim he/she is from the origin of the platform?

3. The velocity v of a particle does not transform particularly nicely under Lorentz transformations. Consider however the rapidity η, defined as

$$\eta = \frac{1}{2}\log\frac{c+v}{c-v}$$

   (a) Show that rapidity is *additive*, in the sense that if a particle is moving along x with rapidity η in the lab frame, and the lab frame is moving with respect to a second frame with rapidity η_0, also along x, then the rapidity of our particle as seen from the second reference frame is $\eta + \eta_0$.

   (b) Now we want to generalize this nice property to more general setups in 3D, e.g. a particle moving along x in the lab frame, which is itself moving along y with respect to a second frame. Argue that there is no vector generalization $\vec{\eta}$ of rapidity that retains the additive property above. That is, one would like to have that under a Lorentz transformation of rapidity $\vec{\eta}_0$, the rapidity $\vec{\eta}$ of a particle transforms into $\vec{\eta}+\vec{\eta}_0$. Argue that this is impossible. [*Hint: Recall that Lorentz transformations do not commute, in the sense that when performing two subsequent Lorentz transformation along different axes, the order in which they are performed matters.*]

4. Two observers move in opposite directions in a circle of radius R with constant angular velocities ω_1 and ω_2. When they first meet, they synchronize their clocks. When they meet again, whose clock will be delayed and by how much?

5. A hydrogenic ion consists of a single electron bound to a nucleus with charge Ze, where e is the elementary charge. For a hydrogenic ion, find the scaling with Z of

   (a) the expectation value of r (the electron-nucleus separation)

   (b) the expectation value of E (the total energy of the electron)

   (c) $|\Psi(r = 0)|^2$ (the probability of finding the electron at the nucleus)

   (d) the fine-structure energy splitting

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 12, 2018
10:00AM to 12:00PM
General Physics (Part I)
Section 5.

Two hours are permitted for the completion of this section of the examination. Choose 4 problems out of the 6 included in this section. (You will not earn extra credit by doing an additional problem). Apportion your time carefully.

Use separate answer booklet(s) for each question. Clearly mark on the answer booklet(s) which question you are answering (e.g., Section 5 (General Physics), Question 2, etc.).

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Good Luck!

1. Consider a star of mass M and radius R. Assume that the star has a uniform density and is composed of Hydrogen gas which is 100% ionized and well virialized. Assume the gas obeys the ideal gas law. The star radiates energy and therefore gradually contracts.

   (a) Estimate the star temperature in terms of mass M and radius R. Is the temperature increasing or decreasing with contracting radius R? How do you explain this behavior?

   (b) What is the electron de Broglie wavelength of the star's gas in terms of temperature?

   (c) Define the critical density of the star ρ_0 as the density at which quantum effects change the pressure. Express ρ_0 in terms of the electron de Broglie wavelength. Will the star continue to contract to $\rho > \rho_0$?

   (d) Using the above results, find an expression for the maximum temperature a star can reach as a function of its mass. Estimate this temperature for a star of mass $M = 2 \times 10^{33}$ g (mass of the sun).

   For your numerical estimates you may find useful:

   Gravitational constant $G = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}
   Planck constant $\hbar = 1.05 \times 10^{-27}$ erg s
   Electron mass $m_e = 0.91 \times 10^{-27}$ g
   Proton mass $m_p = 1.67 \times 10^{-24}$ g
   Classical electron radius $r_e = e^2/m_e c^2 = 2.82 \times 10^{-13}$ cm
   Proton radius $r_p \sim 0.9$ fm$= 0.9 \times 10^{-13}$ cm

2. A system with two nondegenerate energy levels, E_0 and E_1 ($E_1 > E_0 > 0$) is populated by N distinguishable particles at temperature T.

   (a) What is the average energy per particle? Express answer in terms of E_0, E_1 and $\Delta E = E_1 - E_0$.

   (b) What is the average energy per particle as $T \to 0$? Express answer in terms of E_1 and ΔE.

   (c) What is the average energy per particle as $T \to \infty$? Express answer in terms of $E_0 + E_1$ and ΔE.

   (d) What is the specific heat at constant volume, c_V, of this system? Express answer in terms of ΔE.

   (e) Compute c_V in the limits $T \to 0$ and $T \to \infty$ and make a sketch of c_V versus $\Delta E/kT$.

3. This is a problem on the one-dimensional Ising model. Consider a system with $N+1$ spins with the Hamiltonian:

$$H = -J \sum_{i=0}^{N-1} s_{i+1} s_i \tag{1}$$

where each s_i can take on only two possible values: $+1$ or -1. Let us define $G \equiv \beta J$, where β is the usual inverse temperature. Thus, the partition function Z takes the form

$$Z = \sum_{\{s\}} \exp \left[G \sum_{i=0}^{N-1} s_{i+1} s_i \right] \tag{2}$$

where the symbol $\{s\}$ denotes all possible combinations of the values of the spins.

(a) Compute Z. You should be able to express it as an analytic function of G (and N). Hint: it is useful to define a variable $t_i \equiv s_{i+1} s_i$ which also takes the values $+1$ or -1. You might also want to assume first that s_0 is fixed and consider a sum over all possible combinations of $t_0, t_1, ..., t_{N-1}$; then sum over $s_0 = \pm 1$.

(b) What is the free energy F? What is its low temperature ($\beta \to \infty$) limit? Be careful to distinguish between the $J > 0$ (ferromagnetic) and $J < 0$ (anti-ferromagnetic) possibilities.

4. In a system of electrons confined to a two-dimensional plane, the energy of the states of electrons is represented as:

$$E[j, p_x, p_y] = \frac{1}{2m_e}(p_x^2 + p_y^2) + jE_z, \qquad E_z = 0.01 \text{ eV},$$

where jE_z, $j = 0, 1, 2, \ldots$, represents the quantized energy of electron oscillations along the z-direction normal to the (x, y)-plane, and (p_x, p_y) are the components of electron momentum in the (x, y)-plane. The (x, y)-plane has a (large) surface area A.

(a) Evaluate the density of states for each value of the quantum number j.

(b) Assume that the electron density is $n = 10^{12}$ cm^{-2}. Find the chemical potential at temperature $T = 0$ K. Describe qualitatively what happens when the temperature is raised to $T \sim 100\,$K.

Some numerical values:

1 eV $= 1.6 \times 10^{-12}$ erg $= 1.6 \times 10^{-19}$ Joule, and equivalent to about 10000 K
$m_e = 9.1 \times 10^{-28}$ g
$e = 4.8 \times 10^{-10}$ esu$=1.6 \times 10^{-19}$ C
$\hbar = 1.05 \times 10^{-27}$ erg s $= 1.05 \times 10^{-34}$ Joule $\cdot$ s

5. Earth's atmosphere is composed of a wide range of gases including water vapor. Estimate the ratio of the number of water molecules to the total number of gas particles (atoms and molecules) in a cubic meter of air in typical ambient conditions using the following information:

   - Typical ambient temperature $T_A = 293$ K and relative humidity $\rho = 50\%$.
   - Saturation vapor pressure of water approximately doubles with every 10 K rise in temperature.
   - Boiling temperature T_B of water at typical ambient pressure is 373 K.

6. Consider a system of N independent identical harmonic oscillators whose energy is given by

$$E = \sum_{i=1}^{N} \left(m_i + \tfrac{1}{2}\right)\hbar\omega,$$

where $m_i = 0, 1, 2, \ldots$ are the oscillator excitation quantum numbers. The harmonic oscillators describe the motion of atoms in a solid. This model was introduced by Einstein in 1905 to understand thermodynamic properties (e.g. the heat capacity) of solids.

(a) Determine the number of ways, $\Omega(E)$, that this energy can be obtained. Hint: $\Omega(E)$ can be understood as the number of ways of putting $M \equiv \sum_{i=1}^{N} m_i$ indistinguishable balls in N labeled boxes.

(b) Find the entropy S in terms of N and M, assuming N and M are both large.

(c) Using the relation between entropy S, energy E, and temperature T, find T in terms of N and M.

(d) Express E in terms of N and T. Which form does E take in the limiting cases $k_{\mathrm{B}}T \ll \hbar\omega$ and $k_{\mathrm{B}}T \gg \hbar\omega$?

Columbia University
Department of Physics
QUALIFYING EXAMINATION

Friday, January 12, 2018
2:00PM to 4:00PM
General Physics (Part II)
Section 6.

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Good Luck!

1. Suppose you are doing a spectroscopy experiment on a cloud of ^{84}Kr atoms at room temperature and negligible pressure. You want to tune a laser to probe a transition at 811.51 nm (with respect to an atom at rest) by passing the laser through the gas in one direction. To make your measurements, you would like to use an old interferometer you found hidden in the bottom of a drawer in your lab. It has a resolving power of 10^7. You may use the following information: the RMS velocity of ^{84}Kr atoms at room temperature is 300 ms^{-1} and the collision cross-section for ^{84}Kr atoms is $\approx 10^{-15}$ cm^2.

   (a) Estimate the dominant relative uncertainty on the measurement of the line. Suppose you could first cool the cloud to 1 K. Estimate the expected improvement in your results, if any.

   (b) Suppose that for the measurement above, the atoms are first put into a long-lived excited state (i.e. metastable state). The atoms must stay in this state as they freely expand from the excitation chamber to the detection chamber, 50 cm away. If the atoms collide with each other, however, they are likely to de-excite (a process called collisional quenching). Assume that collisions with the walls are negligible. Estimate the maximum pressure of gas that can be safely used in the region between chambers, both for a gas at 300 K and a gas at 1 K.

   (c) Once the atoms enter the detection chamber, they must each be probed for 0.5 s. What is the maximum pressure that can safely be maintained in this chamber, again for a gas at 300 K and at 1 K?

2.

(a) Using classical electricity and magnetism arguments (i.e. Coulombs potential), make an order of magnitude comparison of chemical energy to nuclear fission energy.

(b) Write a primary reaction for uranium fission. State two key features necessary for the production of nuclear energy.

(c) Shortly after detonation the fireball of a uranium fission bomb consists of a sphere of gas of radius 15 meters and temperature of 100 million K. Assuming that the expansion is adiabatic and that the fireball remains spherical, estimate the radius of the ball when the temperature is 3000 K.

(d) Thermonuclear weapons can be three orders of magnitude more powerful than fission bombs. What is the primary explanation for this enormous increase in energy?

3. The peaks of tall mountains are generally much colder than their bases. How much colder?

   Assume our lower atmosphere, heated from below by contact with the warmer earth surface, is slightly convective, because it is made slightly buoyant by that contact. Adiabatic convection results in a variation of temperature, T and pressure, P with altitude, h. Evaluate either $T(h)$ or $P(h)$ in terms of $T(0)$ or $P(0)$ using the following:

   The equation of state of air is $PV = RT$, where R is Boltzmann constant multiplied by the Avogadro number N_A. Air heat capacity c_V (per N_A molecules) is from rigid diatomic molecules of average mass $\bar{m}$ with classically calculated translational and rotational energies (but no excitable vibrational or electronic ones).

4. Neutron stars form when a solar mass is compressed to a radius $R \sim 10$ km. Estimate the maximum spin rate of a neutron star. Express your answer in revolutions per second. Recall that the radius of Earths orbit around the sun is $r_E \approx 1.5 \times 10^8$ km.

5. Typical dust particles in the Solar System have mass density ρ. They experience both gravitational and radiation pressure effects of the Sun (mass $M = 2 \times 10^{30}$ kg, emitted light power $P = 4 \times 10^{26}$ W).

   (a) Are the smaller or the larger dust particles more likely to be ejected from the Solar System?

   (b) Estimate, symbolically, the critical radius R of a particle that would not be ejected from the Solar System.

   (c) Provide a numerical estimate to part (b).

6. The MicroBooNE Liquid Argon Time Projection Chamber (LAr TPC) detector is a large rectangular volume with dimensions shown in the figure. It is nominally filled with liquid argon. Ionization electrons, liberated by a charged particle propagating through the liquid argon along the particle's path, drift under an applied uniform electric field toward a plane of anode wires that lie in the shaded plane in the figure. The wires are strung vertically from the top to the bottom of the TPC, and spaced 3 mm apart along the length (L) of the TPC. Answer the following questions:

   (a) During the filling of the TPC, the liquid level h changed with a constant rate dh/dt. The potential difference across the TPC was held fixed at V_0. The resulting change in effective capacitance in the TPC resulted in increasing noise levels on the TPC anode wires as a function of increasing liquid level. Calculate the rate of change of effective TPC capacitance, dC/dt.

   (b) A minimum ionizing particle (MIP) traverses the length of the TPC (see figure) near the bottom when the detector is half full. The particle loses energy due to ionization in the argon at a rate of 2 MeV/cm. The ionization energy of argon is 23.6 eV. You may assume that half of the ionization charge liberated is lost during the drift. The noise level on a single TPC wire for an empty detector, measured in Equivalent Noise Charge (ENC), is 400 electrons; when the detector is full, the ENC is 450 electrons. Calculate the signal-to-noise ratio, on a single TPC wire, under these conditions.